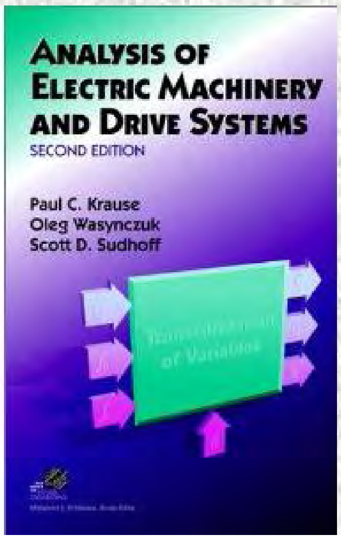
بسم الله الرحمن الرحیم

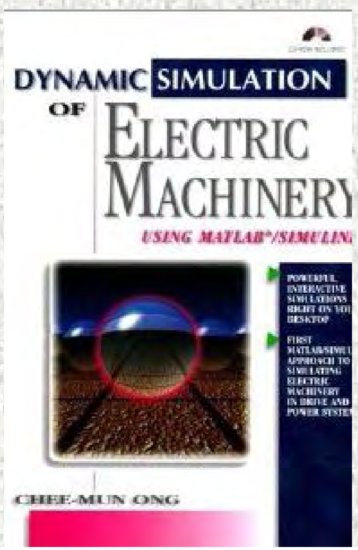
تئوری جامع ماشینهای الکتریکی

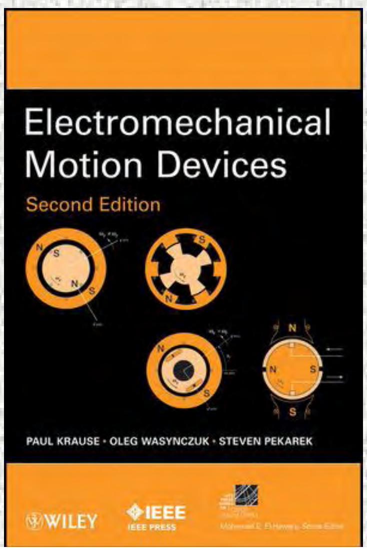
مدرس: دکتر عباس کتابی

مراجع درس:

- “Analysis of Electric Machinery and Drive Systems”, Third Edition, By: P. C. Krause, O. Wasynczuk and S. D. Sudhoff, IEEE Press, 2013.
- “Electromechanical Motion Devices”, Second Edition, By: P. C. Krause, O. Wasynczuk and S. Pekarek, Wiley-IEEE Press, 2012.
- “Dynamic simulation of Electric Machinery using MATLAB”, by: Chee-Mun Ong, Prentice Hall PTR, 1997







نحوه ارزشیابی:

تکالیف: 2 نمره

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مطالب درس:

- Reference Frame Theory
- 3-Phase Induction Machines
- Synchronous Machines
- Machine Equations in Operational Impedances and Time Constants
- Linearized Machine Equations
- Reduced Order Machine Equations

Reference Frame Theory

➢ **Power of Reference Frame Theory:**

- Eliminates Rotor Position Dependence inductances
- Transforms Nonlinear Systems to Linear Systems for Certain Cases
- Fundamental Tool For Development of Equivalent Circuits
- Can Be Used to Make AC Quantities Become DC Quantities
- Framework of Most Controllers

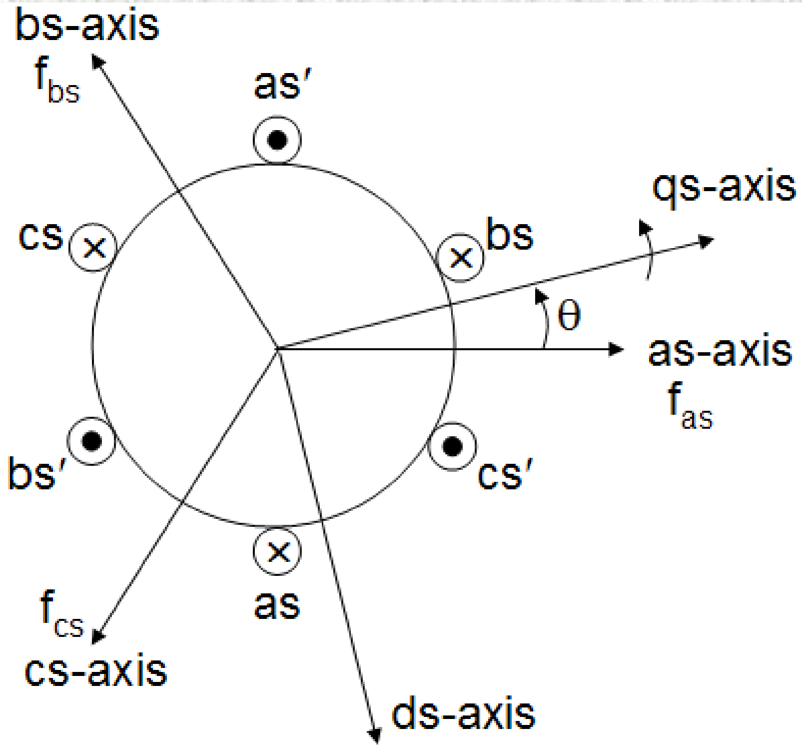
Reference Frame Theory

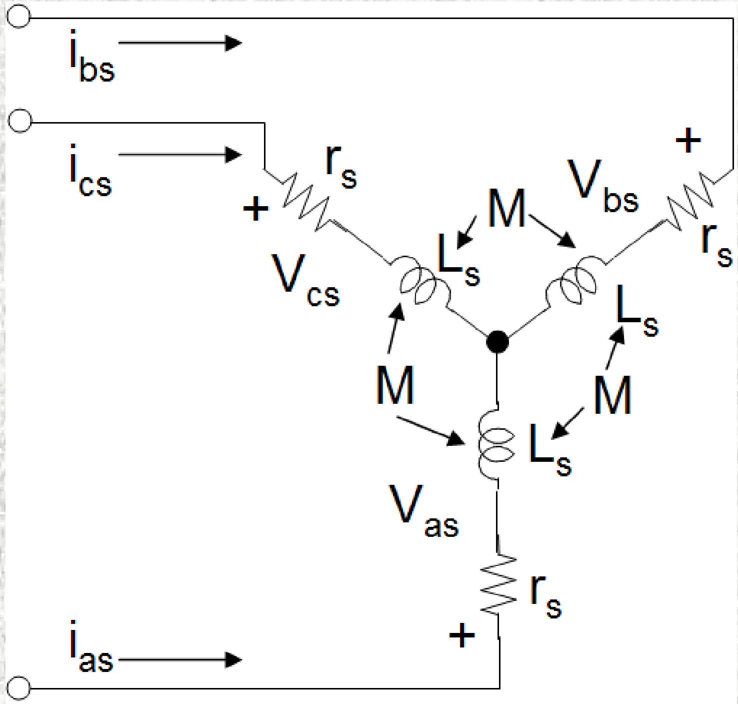
➢**History of Reference Frame Theory**

- 1929: Park's Transformation
  - ☞ Synchronous Machine; Rotor Reference Frame
- 1938: Stanley
  - ☞ Induction Machine; Stationary Reference Frame
- 1951:Kron
  - ☞ Induction Machine; Synchronous Reference Frame
- 1957: Brereton
  - ☞ Induction Machine; Rotor Reference Frame
- 1965: Krause
  - ☞ Arbitrary Reference Frame

Arbitrary Reference Frame

- Consider stator winding of a 2-pole 3-phase symmetrical machine





Arbitrary Reference Frame

- Synchronous and induction machine **inductances** are **functions of the rotor speed**, therefore the coefficients of the differential equations (voltage equations) which describe the behavior of these machines are **time-varying**.

- A change of variables can be used to reduce the complexity of machine differential equations, and represent these equations in another reference frame with **constant coefficients**.

Arbitrary Reference Frame

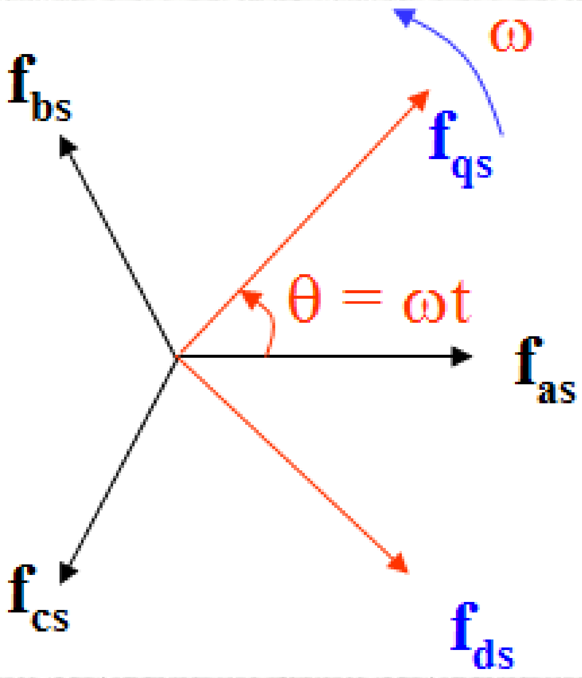
- A change of variables which formulates a transformation of the 3-phase variables of stationary circuit elements to the arbitrary reference frame may be expressed

$$\mathbf{f}_{qd0s} = \mathbf{K}_s \mathbf{f}_{abcs}$$

$$\mathbf{K}_s = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix},$$

$$where, \quad (\mathbf{f}_{qd0s})^T = \begin{bmatrix} f_{qs} & f_{ds} & f_{0s} \end{bmatrix},$$

$$(\mathbf{f}_{abcs})^T = \begin{bmatrix} f_{as} & f_{bs} & f_{cs} \end{bmatrix}, \qquad \theta = \int_0^t \omega(t)dt + \theta(0).$$

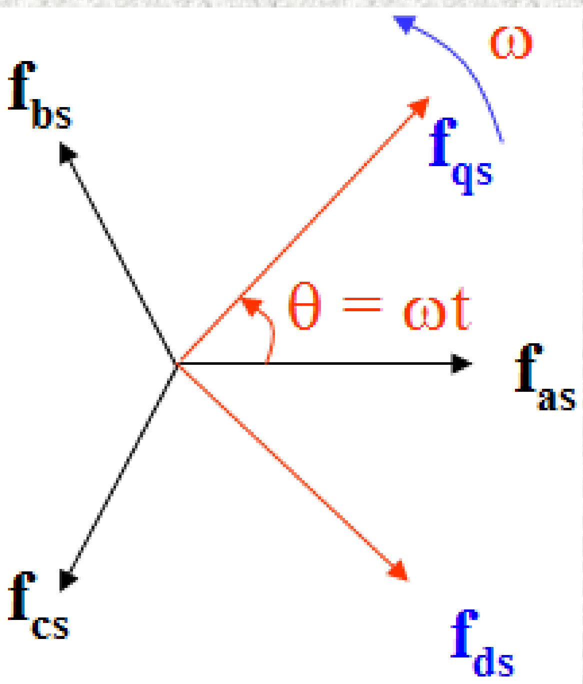


Arbitrary Reference Frame

$$\left(\mathbf{K}_s\right)^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix}.$$

- "f" can represent either voltage, current, or flux linkage.
- "s" indicates the variables, parameters and transformation associated with stationary circuits.
- "ω" represent the speed of reference frame.

Arbitrary Reference Frame



- $\omega=0$: Stationary reference frame.
- $\omega=\omega_e$: synchronoulsy rotaing reference frame.
- $\omega=\omega_r$: rotor reference frame (i.e., the reference frame is fixed on the rotor).

Arbitrary Reference Frame

- f_{as}, f_{bs} and f_{cs} may be thought of as the direction of the magnetic axes of the stator windings.

- f_{qs} and f_{ds} can be considered as the direction of the magnetic axes of the "new" fictious windings located on qs and ds axis which are created by the change of variables.

- Power Equations:

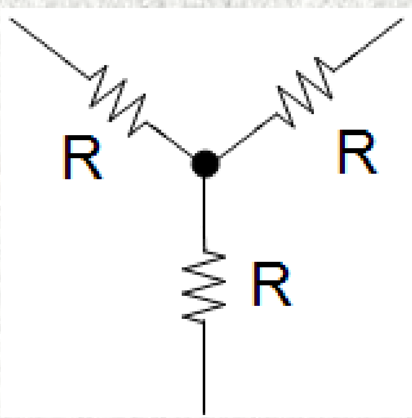
$$P_{abcs} = V_{as} i_{as} + V_{bs} i_{bs} + V_{cs} i_{cs}$$

$$P_{qd0s} = P_{abcs} = \frac{3}{2}\left(V_{qs} i_{qs} + V_{ds} i_{ds} + 2V_{0s} i_{0s}\right)$$

Arbitrary Reference Frame

- Stationary circuit variables transformed to the arbitrary reference frame.
- Resistive elements: For a 3-phase resistive circuit,

$$V_{abcs} = \bar{r}_s i_{abcs} \qquad \bar{r}_s = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix}$$



$$i_{abcs} = (\mathbf{K}_s)^{-1} i_{qd0s} \qquad V_{abcs} = (\mathbf{K}_s)^{-1} V_{qd0s}$$

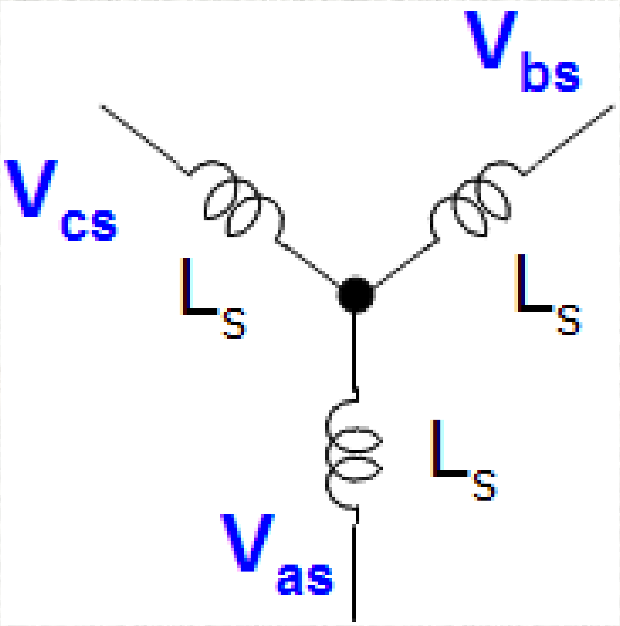
$$(\mathbf{K}_s)^{-1} V_{qd0s} = \bar{r}_s (\mathbf{K}_s)^{-1} i_{qd0s}$$

$$V_{qd0s} = (\mathbf{K}_s)\bar{r}_s (\mathbf{K}_s)^{-1} i_{qd0s} \quad , (\mathbf{K}_s)\bar{r}_s (\mathbf{K}_s)^{-1} = \bar{r}_s$$

$$V_{qd0s} = \bar{r}_s i_{qd0s}$$

Arbitrary Reference Frame

- Inductive elements: For a 3-phase inductive circuit,



$$V_{abcs} = p\lambda_{abcs},$$

$$where,\ p = \frac{d}{dt},$$

$$\lambda_{abcs} = \mathbf{L}_s i_{abcs} = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_s \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

Arbitrary Reference Frame

- In terms of the substitute variables, we have

$$\mathbf{V}_{qd0s} = \mathbf{K}_s \cdot p\left[\mathbf{K}_s^{-1}\lambda_{qd0s}\right] = \mathbf{K}_s \cdot p\left[\mathbf{K}_s^{-1}\right]\lambda_{qd0s} + \mathbf{K}_s \cdot \left[\mathbf{K}_s^{-1}\right]p\lambda_{qd0s}$$

$$where,\ p\left[\mathbf{K}_s^{-1}\right] = \omega \cdot \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\sin(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & 0 \\ -\sin(\theta + \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) & 0 \end{bmatrix}$$

- After some work, we can show that

$$\mathbf{K}_s \cdot p\left[\mathbf{K}_s^{-1}\right] = \omega \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Arbitrary Reference Frame

$$V_{qd0s} = \underbrace{\mathbf{K}_s p[(\mathbf{K}_s)^{-1}]\lambda_{qd0s}}_{\omega\lambda_{dqs}} + \underbrace{\mathbf{K}_s (\mathbf{K}_s)^{-1} p\lambda_{qd0s}}_{p\lambda_{qd0s}}$$

$$V_{qd0s} = \omega\lambda_{dqs} + p\lambda_{qd0s}$$

$$\text{where,} \quad (\lambda_{dqs})^T = \begin{bmatrix} \lambda_{ds} & -\lambda_{qs} & 0 \end{bmatrix}$$

- Vector equation V_{qd0s} can be expressed as

$$V_{qs} = \omega\lambda_{ds} + p\lambda_{qs}$$

$$V_{ds} = -\omega\lambda_{qs} + p\lambda_{ds}$$

$$V_{0s} = p\lambda_{0s}$$

where "$\omega\lambda_{ds}$" term and "$\omega\lambda_{qs}$" term are referred to as a "speed voltage" with the speed being the angular velocity of the arbitrary reference frame.

Arbitrary Reference Frame

- When the reference frame is fixed in the stator, that is, the stationary reference frame (ω=0), the voltage equations for the three-phase circuit become the familiar time rate of change of flux linkage in abcs reference frame

- For the three-phase circuit shown, L_s is a diagonal matrix, and

$$\lambda_{abcs} = \mathbf{L}_s i_{abcs}$$

$$\lambda_{qd0s} = \mathbf{K}_s \mathbf{L}_s \mathbf{K}_s^{-1} i_{qd0s} = \mathbf{L}_s i_{qd0s}$$

Arbitrary Reference Frame

- For the three-phase induction or synchronous machine, L_s matrix is expressed as

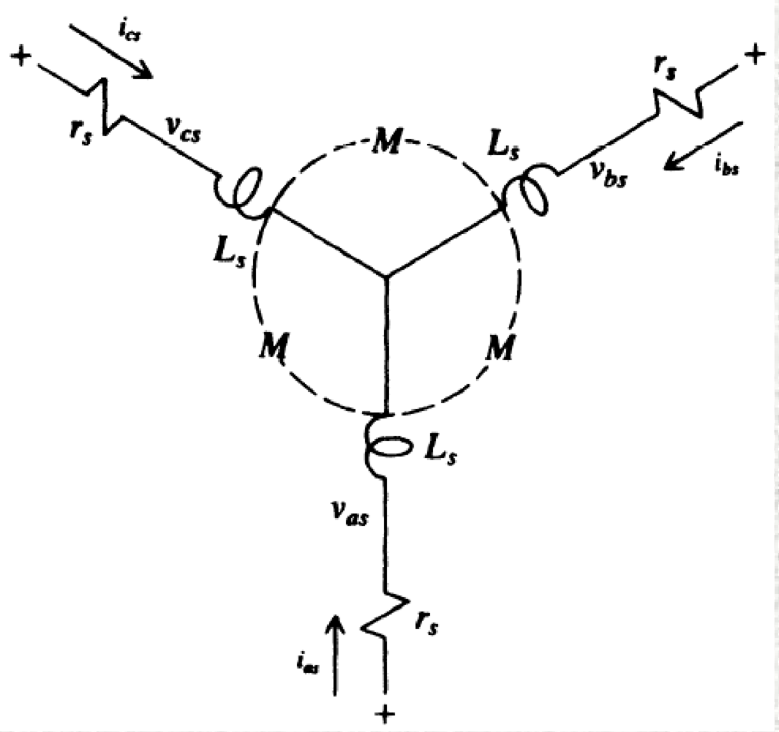
$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix}$$

where, L_{ls}: leakage inductance, L_{ms}: magnetizing inductance

$$\mathbf{K}_s \mathbf{L}_s (\mathbf{K}_s)^{-1} = \begin{bmatrix} \mathbf{L}_{ls} + \frac{3}{2}\mathbf{L}_{ms} & 0 & 0 \\ 0 & \mathbf{L}_{ls} + \frac{3}{2}\mathbf{L}_{ms} & 0 \\ 0 & 0 & \mathbf{L}_{ls} \end{bmatrix}$$

Arbitrary Reference Frame

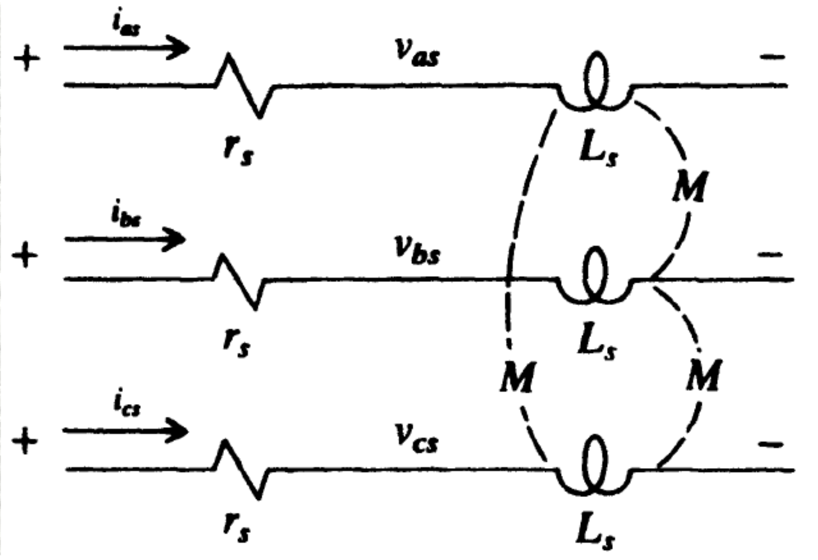
- Consider the stator windings of a symmetrical induction or round rotor synchronous machine shown below



$$\mathbf{r}_s = diag\begin{bmatrix} r_s & r_s & r_s \end{bmatrix}$$

$$\mathbf{L}_s = \begin{bmatrix} L_s & M & M \\ M & L_s & M \\ M & M & L_s \end{bmatrix} \quad L_s = L_{ls} + L_{ms}$$

$$M = -\frac{1}{2} L_{ms}$$



Arbitrary Reference Frame

- For each phase voltage, we write the following equations,

$$V_{as} = r_s i_{as} + p\lambda_{as}$$
$$V_{bs} = r_s i_{bs} + p\lambda_{bs},$$
$$V_{cs} = r_s i_{cs} + p\lambda_{cs}$$

$$V_{qd0s} = \mathbf{K}_s V_{abcs}$$
$$i_{qd0s} = \mathbf{K}_s i_{abcs}$$
$$\lambda_{qd0s} = \mathbf{K}_s \lambda_{abcs}$$
$$\lambda_{abcs} = \mathbf{L}_s i_{abcs}$$

- In vector form,

$$V_{abcs} = \mathbf{r}_s i_{abcs} + p\lambda_{abcs},$$

Multiplying by $\mathbf{K}_s$

$$\mathbf{K}_s V_{abcs} = \mathbf{K}_s \mathbf{r}_s i_{abcs} + \mathbf{K}_s p\lambda_{abcs}$$

Arbitrary Reference Frame

دانشگاه کاشان

- Replace i_{abcs} and λ_{abcs} using the transformation equations,

$$\mathbf{K}_s V_{abcs} = \mathbf{K}_s \mathbf{r}_s (\mathbf{K}_s^{-1} i_{qd0s}) + \mathbf{K}_s p(\mathbf{K}_s^{-1} \lambda_{qd0s})$$

$$V_{qd0s} = \mathbf{r}_s i_{qd0s} + \overline{\omega}\overline{\lambda}_{qd0s}$$

or

$$V_{qs} = r_s i_{qs} + \omega\lambda_{ds} + p\lambda_{qs}$$

$$V_{ds} = r_s i_{ds} - \omega\lambda_{qs} + p\lambda_{ds}$$

$$V_{0s} = r_s i_{0s} + p\lambda_{0s}$$

$$\mathbf{K}_s \mathbf{L}_s (\mathbf{K}_s)^{-1} = \begin{bmatrix} L_s - M & 0 & 0 \\ 0 & L_s - M & 0 \\ 0 & 0 & L_s + 2M \end{bmatrix}$$

where, $\overline{\omega} = \omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

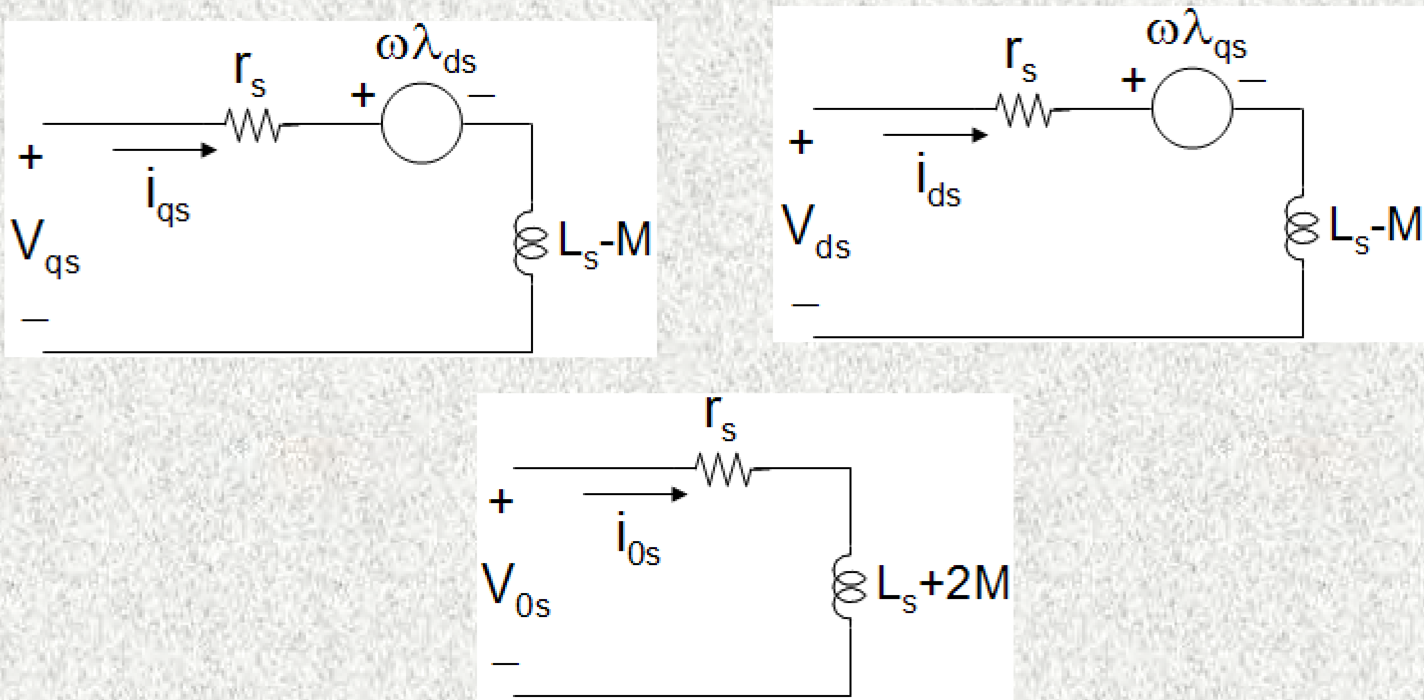
$$\lambda_{qs} = (L_s - M) i_{qs}$$

$$\lambda_{ds} = (L_s - M) i_{ds}$$

$$\lambda_{0s} = (L_s + 2M) i_{0s}$$

Commonly Used Reference Frames

- Our equivalent circuit in arbitrary reference frame can be represented as



Commonly used reference frame

Commonly Used Reference Frames

- ω=unspecified: stationary circuit variables referred to the arbitrary reference frame. The variables are referred to as f_{qd0s} or f_{qs}, f_{ds} and f_{0s} and transformation matrix is designated as K_s.

- ω=0: stationary circuit variables referred to the stationary reference frame. The variables are referred to as f^s_{qd0s} or f^s_{qs}, f^s_{ds} and f^s_{0s} and transformation matrix is designated as K^s_s.

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Commonly Used Reference Frames

- $\omega = \omega_r$: stationary circuit variables referred to the reference frame fixed in the rotor. The variables are referred to as f^r_{qd0s} or f^r_{qs}, f^r_{ds} and f^r_{0s} and transformation matrix is designated as K^r_s.

- $\omega = \omega_e$: stationary circuit variables referred to the synchronously rotating reference frame. The variables are referred to as f^e_{qd0s} or f^e_{qs}, f^e_{ds} and f^e_{0s} and transformation matrix is designated as K^e_s.

Commonly Used Reference Frames

- Representation

f_{qd0s}^{s} → Stationary reference frame; → q-d axes of stator variables

f_{qd0s}^{r} → Reference frame fixed on the rotor with speed of ω_r; → q-d axes of stator variables, $\theta_r = \int_0^t \omega_r(t)dt$

f_{qd0s}^{e} → Synchronously rotating reference frame; → q-d axes of stator variables, $\theta_e = \int_0^t \omega_e(t)dt$

Transformation of a Balanced Set

- Consider a 3-phase circuit which is excited by a balanced 3-phase voltage set. Assume the balanced set is a set of equal amplitude sinusoidal quantities which are displaced by 120°.

$$f_{as} = \sqrt{2} f_s \cos\theta_{ef}$$

$$f_{bs} = \sqrt{2} f_s \cos(\theta_{ef} - \frac{2\pi}{3})$$

$$f_{cs} = \sqrt{2} f_s \cos(\theta_{ef} + \frac{2\pi}{3})$$

$$f_{as} + f_{bs} + f_{cs} = 0 \text{ (balanced set)}$$

$$\theta_{ef} = \int_0^t \omega_e(t)dt + \theta_{ef}(0)$$

- θ_{ef}: Angular position of each electrical variable (voltage, current, and flux linkage) is θ_{ef} with the f subscript used to denote the specific electrical variable.

Transformation of a Balanced Set

- θ_e: Angular position of the synchronously rotating reference frame is θ_e.
- θ_e and θ_{ef} differ only in the zero position $\theta_e(0)$ and $\theta_{ef}(0)$, since each has the same angular velocity of ω_e.
- f_{as}, f_{bs} and f_{cs} can be transformed to the arbitrary reference frame,

$$\bar{f}_{qd0s} = \overline{\mathbf{K}}_s \, \bar{f}_{abcs}$$

Transformation of a Balanced Set

- After transformation, we will have,

$$f_{qs} = \sqrt{2} f_s \cos(\theta_{ef} - \theta)$$

$$f_{ds} = -\sqrt{2} f_s \sin(\theta_{ef} - \theta)$$

$$f_{0s} = 0$$

- *qs* and *ds* variables form a balanced 2-phase set in all reference frames except when $\omega = \omega_e$,

$$f^{e}{}_{qs} = \sqrt{2} f_s \cos\left[\theta_{ef}(0) - \theta_e(0)\right]$$

$$f^{e}{}_{ds} = -\sqrt{2} f_s \sin\left[\theta_{ef}(0) - \theta_e(0)\right]$$

- In qs^e and ds^e reference frame, sinusoidal quantities appear as constant dc quantities.

Balanced Steady-State Phasor Relationships

- For balanced steady-state conditions ω_e is constant and sinusoidal quantities can be represented as phasor variables.

$$F_{as} = \sqrt{2}F_s \cos\left[\omega_e t + \theta_{ef}(0)\right] = \mathrm{Re}\left[\sqrt{2}F_s e^{j\theta_{ef}(0)} e^{j\omega_e t}\right]$$

$$F_{bs} = \sqrt{2}F_s \cos\left[\omega_e t + \theta_{ef}(0) - \frac{2\pi}{3}\right] = \mathrm{Re}\left[\sqrt{2}F_s e^{j\left(\theta_{ef}(0) - \frac{2\pi}{3}\right)} e^{j\omega_e t}\right]$$

$$F_{bs} = \sqrt{2}F_s \cos\left[\omega_e t + \theta_{ef}(0) + \frac{2\pi}{3}\right] = \mathrm{Re}\left[\sqrt{2}F_s e^{j\left(\theta_{ef}(0) + \frac{2\pi}{3}\right)} e^{j\omega_e t}\right]$$

Balanced Steady-State Phasor Relationships

- Balanced steady-state qs-ds variables are,

$$F_{qs} = \sqrt{2}F_s \cos\left[(\omega_e - \omega)t + \theta_{ef}(0) - \theta(0)\right]$$
$$= \mathrm{Re}\left[\sqrt{2}F_s e^{j\left(\theta_{ef}(0)-\theta(0)\right)} e^{j(\omega_e-\omega)t}\right]$$

$$F_{ds} = -\sqrt{2}F_s \sin\left[(\omega_e - \omega)t + \theta_{ef}(0) - \theta(0)\right]$$
$$= \mathrm{Re}\left[j\sqrt{2}F_s e^{j\left(\theta_{ef}(0)-\theta(0)\right)} e^{j(\omega_e-\omega)t}\right]$$

- f_{as} phasor can be expressed as

$$\tilde{F}_{as} = F_s e^{j\theta_{ef}(0)}$$

Balanced Steady-State Phasor Relationships

- For arbitrary reference frame ($\omega \neq \omega_e$),

$$\tilde{F}_{qs} = F_s e^{j\left(\theta_{ef}(0) - \theta(0)\right)}, \quad \tilde{F}_{ds} = j\tilde{F}_{qs}$$

- Selecting $\theta(0)=0$,

$$\tilde{F}_{as} = \tilde{F}_{qs}$$

- Thus, in all asynchronously rotating reference frame ($\omega \neq \omega_e$) with $\theta(0)=0$, the phasor representing the *as* variables is equal to the phasor representing the *qs* variables.

Balanced Steady-State Phasor Relationships

- In the synchronously rotating reference frame $\omega=\omega_e$, F^e_{qs} and F^e_{ds} can be expressed as

$$F^e{}_{qs} = \mathrm{Re}\left[\sqrt{2}F_s e^{j\left(\theta_{ef}(0)-\theta(0)\right)}\right]$$

$$F^e{}_{ds} = \mathrm{Re}\left[j\sqrt{2}F_s e^{j\left(\theta_{ef}(0)-\theta(0)\right)}\right]$$

- Let $\theta_e(0)=0$, then

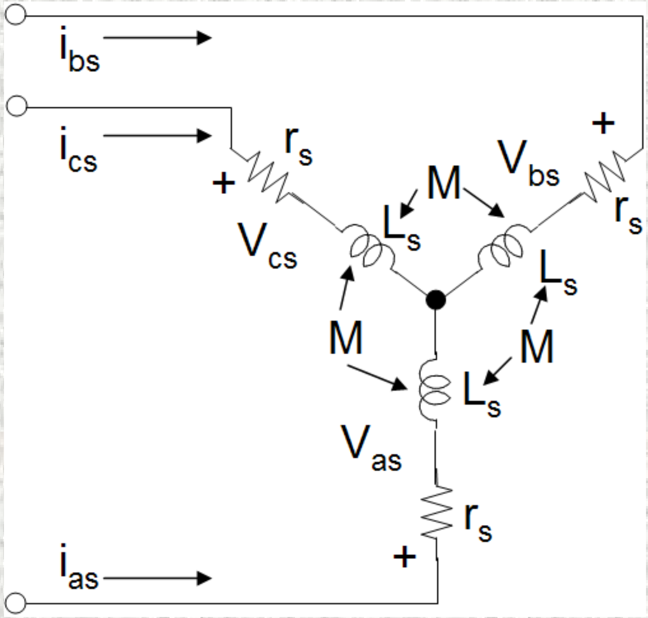
$$F^e{}_{qs} = \sqrt{2}F_s\cos(\theta_{ef}(0)),\ F^e{}_{ds} = -\sqrt{2}F_s\sin(\theta_{ef}(0))$$

$$\sqrt{2}\tilde{F}_{as} = F^e{}_{qs} - jF^e{}_{ds}$$

since, $\tilde{F}_{as} = F_s e^{j(\theta_{ef}(0))} = F_s\cos(\theta_{ef}(0)) + jF_s\sin(\theta_{ef}(0))$

Balanced Steady-State Phasor Relationships

- Consider the stator winding of a symmetrical induction machine.



- Assume the stator winding is excited by a balanced 3-phase sinusoidal voltage set.

Balanced Steady-State Phasor Relationships

- For phase as, we will have $\left(p = \frac{d}{dt} \right)$

$$V_{as} = r_s i_{as} + L_s p i_{as} + M p i_{bs} + M p i_{cs}$$

- For balanced conditions

$$V_{as} + V_{bs} + V_{cs} = 0, \quad i_{as} + i_{bs} + i_{cs} = 0$$

$$M p i_{as} = -M p (i_{bs} + i_{cs}), \quad V_{as} = r_s i_{as} + (L_s - M) p i_{as}$$

- For steady-state conditions, p = jω_e

$$\tilde{V}_{as} = r_s \tilde{I}_{as} + \left[(L_s - M) j\omega_e \right] \tilde{I}_{as}$$

Balanced Steady-State Phasor Relationships

- *qs* and *ds* voltage equations in the arbitrary reference frame can be written as

$$V_{qs} = r_s i_{qs} + \omega\lambda_{ds} + p\lambda_{qs}$$

$$V_{ds} = r_s i_{ds} - \omega\lambda_{qs} + p\lambda_{ds}$$

$$\lambda_{qs} = (L_s - M)i_{qs}, \quad \lambda_{ds} = (L_s - M)i_{ds}$$

- Let $\omega=\omega_e$, then

$$V^e{}_{qs} = r_s i^e{}_{qs} + \omega_e\lambda^e{}_{ds} + p\lambda^e{}_{qs}$$

$$V^e{}_{ds} = r_s i^e{}_{ds} - \omega_e\lambda^e{}_{qs} + p\lambda^e{}_{ds}$$

$$\lambda^e{}_{qs} = (L_s - M)i^e{}_{qs}, \quad \lambda^e{}_{ds} = (L_s - M)i^e{}_{ds}$$

Balanced Steady-State Phasor Relationships

- For balanced steady-state conditions, the variables in the synchronously rotating reference frame are constants, therefore $p\lambda^{e}_{qs}$ and $p\lambda^{e}_{ds}$ are zero. Therefore, the above can be expressed as

$$V^{e}_{qs} = r_s I^{e}_{qs} + \omega_e (L_s - M) I^{e}_{ds}$$

$$V^{e}_{ds} = r_s I^{e}_{ds} - \omega_e (L_s - M) I^{e}_{qs}$$

- Recall $\sqrt{2}\tilde{F}_{as} = F^{e}_{qs} - jF^{e}_{ds}$

- Thus, $\sqrt{2}\tilde{V}_{as} = V^{e}_{qs} - jV^{e}_{ds}$

Balanced Steady-State Phasor Relationships

$$\sqrt{2}\tilde{V}_{as} = r_s I^e{}_{qs} + \omega_e (L_s - M) I^e{}_{ds} - j\left[r_s I^e{}_{ds} + \omega_e (L_s - M) I^e{}_{qs}\right]$$

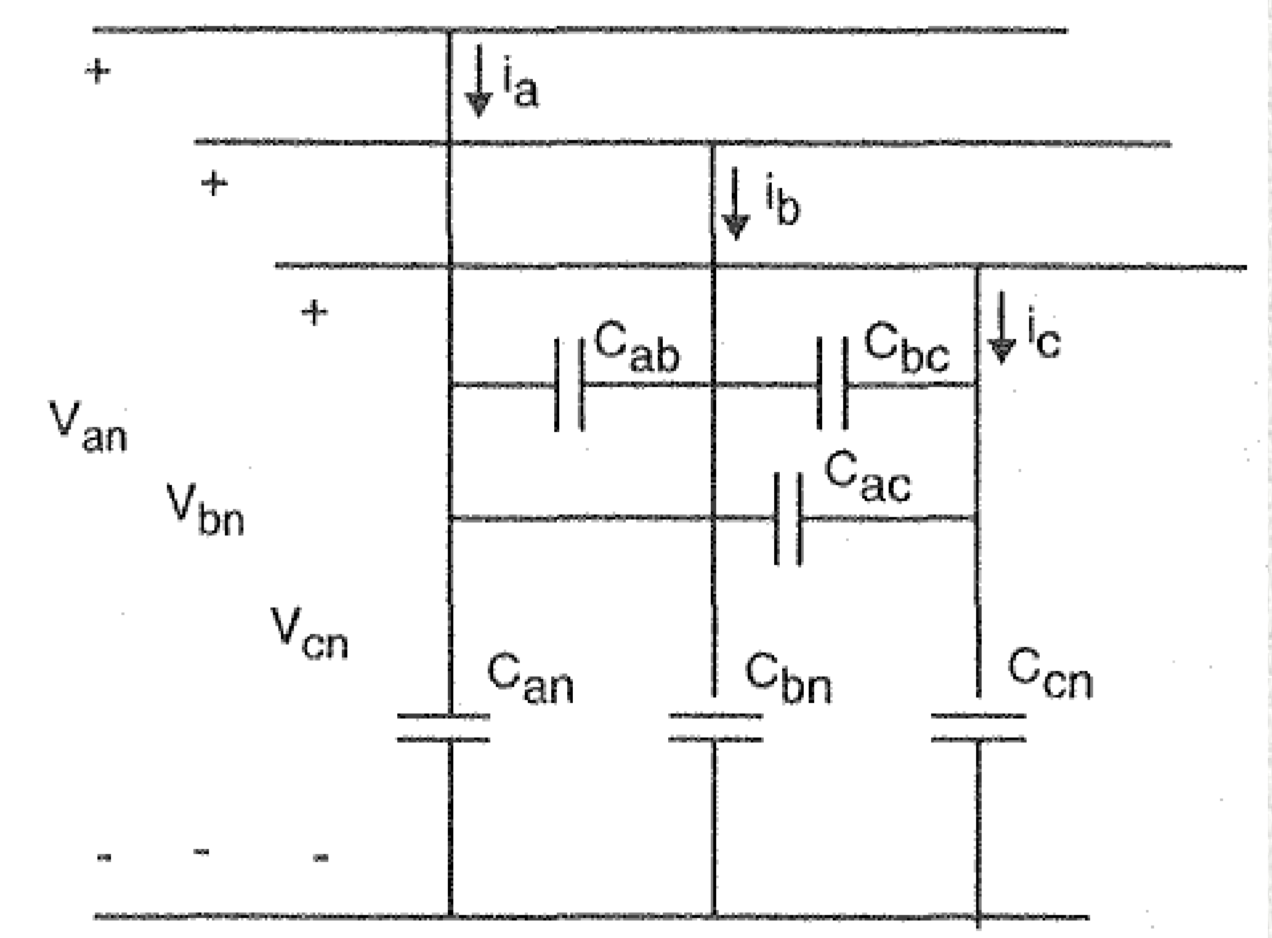
- Now

$$\sqrt{2}\tilde{I}_{as} = I^e{}_{qs} - jI^e{}_{ds}$$
$$j\sqrt{2}\tilde{I}_{as} = I^e{}_{ds} + jI^e{}_{qs}$$

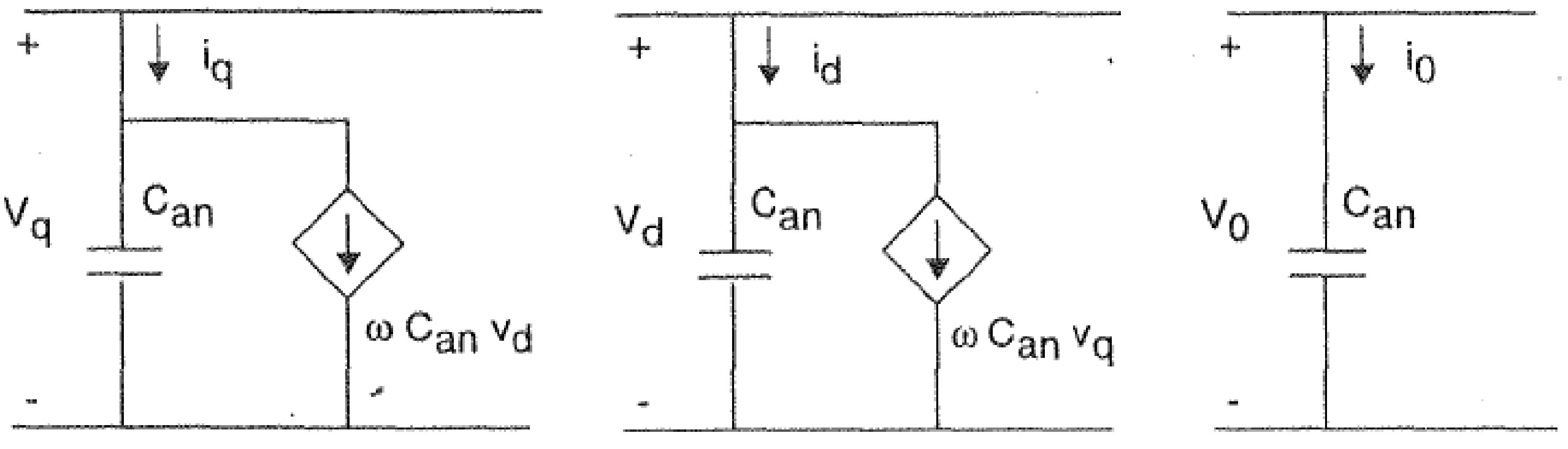
- Substituting in the above equation, we will have

$$\tilde{V}_{as} = \left[r_s + j\omega_e (L_s - M)\right]\tilde{I}_{as}$$

▪ qd0 Transformation to Shunt Capacitances



▪ qd۰ Transformation to Shunt Capacitances



(a) q-axis circuit (b) d-axis circuit (c) 0-axis circuit

Variables Observed From Several Frames of Reference

- Suppose Following Voltages Applied to 3 Phase Wye-Connected RL Circuit:

  - $v_{as} = \sqrt{2}V_s \cos\omega_e t$
  - $v_{bs} = \sqrt{2}V_s \cos\left(\omega_e t - \frac{2\pi}{3}\right)$
  - $v_{cs} = \sqrt{2}V_s \cos\left(\omega_e t + \frac{2\pi}{3}\right)$

Variables Observed From Several Frames of Reference

- Using Basic Circuit Analysis Techniques

$$i_{as} = \frac{\sqrt{2}V_s}{|Z_s|}[-e^{-t/\tau}\cos\alpha + \cos(\omega_e t - \alpha)]$$

$$i_{bs} = \frac{\sqrt{2}V_s}{|Z_s|}\left[-e^{-t/\tau}\cos\left(\alpha + \frac{2\pi}{3}\right) + \cos\left(\omega_e t - \alpha - \frac{2\pi}{3}\right)\right]$$

$$i_{cs} = \frac{\sqrt{2}V_s}{|Z_s|}\left[-e^{-t/\tau}\cos\left(\alpha - \frac{2\pi}{3}\right) + \cos\left(\omega_e t - \alpha + \frac{2\pi}{3}\right)\right]$$

$$Z_s = r_s + j\omega_e L_s$$

$$\tau = \frac{L_s}{r_s}$$

$$\alpha = \tan^{-1}\frac{\omega_e L_s}{r_s}$$

Variables Observed From Several Frames of Reference

- Transforming to the Synchronous Reference Frame:

  - $$i_{qs} = \frac{\sqrt{2}V_s}{|Z_s|}\{-e^{-t/\tau}\cos(\omega t - \alpha) + \cos[(\omega_e - \omega)t - \alpha]\}$$
  - $$i_{ds} = \frac{\sqrt{2}V_s}{|Z_s|}\{-e^{-t/\tau}\sin(\omega t - \alpha) - \sin[(\omega_e - \omega)t - \alpha]\}$$

Variables Observed From Several Frames of Reference

- In Stationary Reference Frame

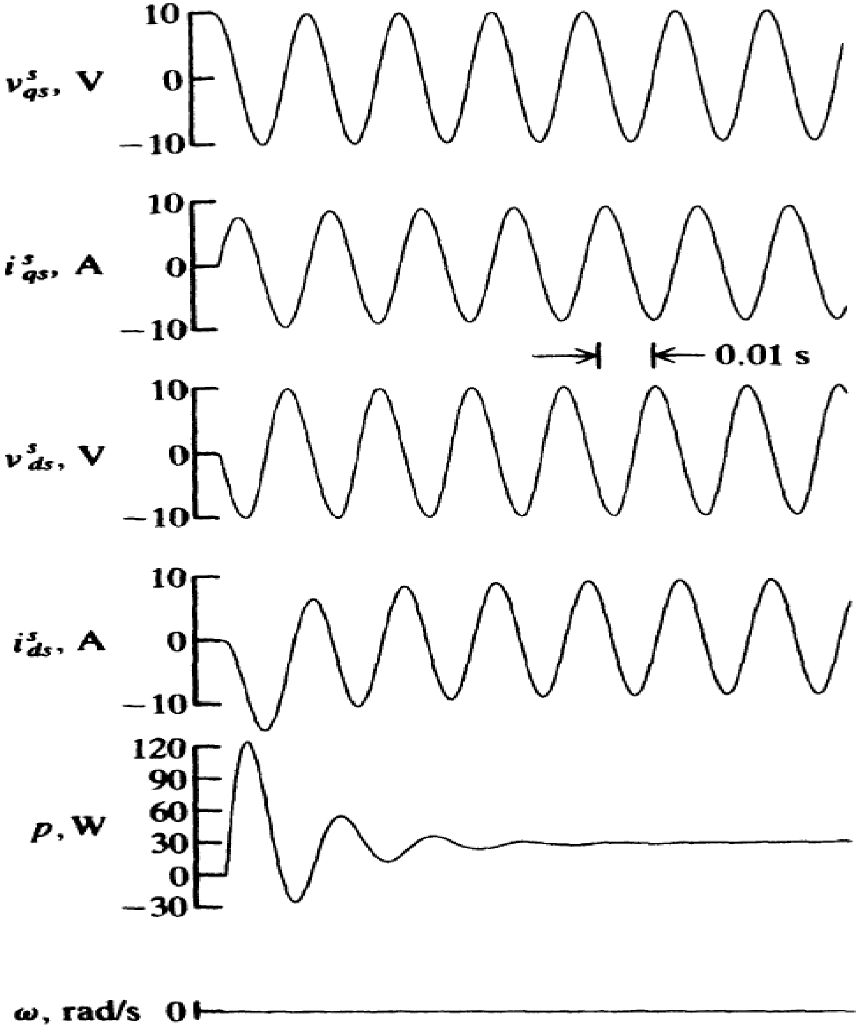
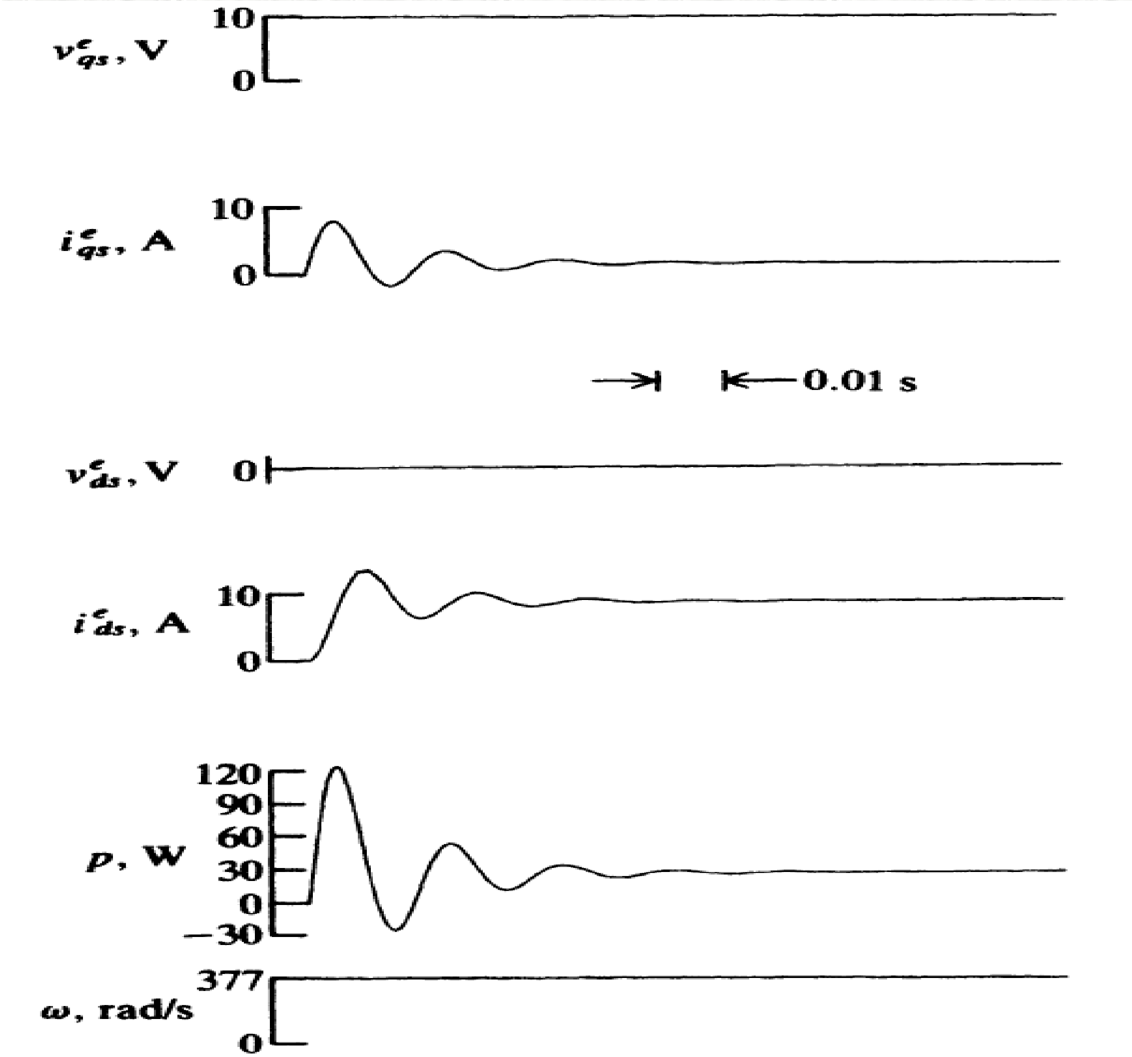


Figure 3.10-1 Variables of a stationary 3-phase system in the stationary reference frame.

Variables Observed From Several Frames of Reference

- In Synchronous Reference Frame



Variables Observed From Several Frames of Reference

- In Strange Reference Frame

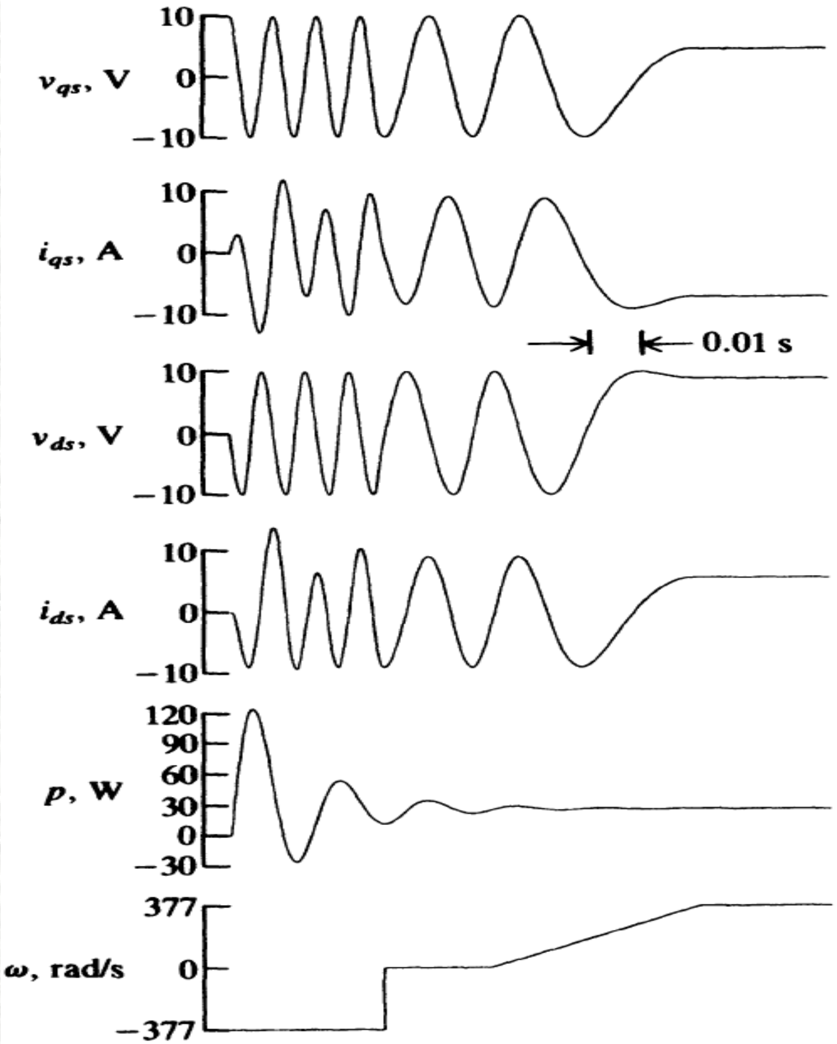


Figure 3.10-3 Variables of a stationary 3-phase system. First with $\omega = -\omega_e$, then ω is stepped to zero followed by a ramp change in reference frame speed to $\omega = \omega_e$.

Transformation Between Reference Frames

- To transform the variables from *x* *to* *y* reference frame :

$$\mathbf{f}^{y}_{qd0s} = {}^{x}\mathbf{K}^{y}\mathbf{f}^{x}_{qd0s}$$

$$\mathbf{f}^{x}_{qd0s} = \mathbf{K}^{x}_{s}\mathbf{f}_{abcs}$$

$\Rightarrow$

$$\mathbf{f}^{y}_{qd0s} = {}^{x}\mathbf{K}^{y}\mathbf{K}^{x}_{s}\mathbf{f}_{abcs}$$

$$\mathbf{f}^{y}_{qd0s} = \mathbf{K}^{y}_{s}\mathbf{f}_{abcs}$$

$\Rightarrow$

$${}^{x}\mathbf{K}^{y}\mathbf{K}^{x}_{s} = \mathbf{K}^{y}_{s}$$

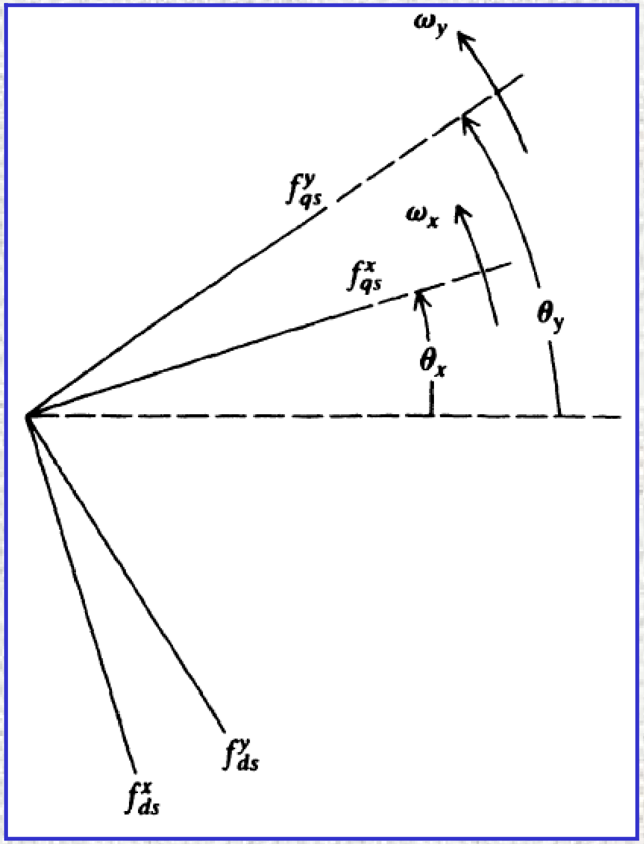
$\Rightarrow$

$${}^{x}\mathbf{K}^{y} = \mathbf{K}^{y}_{s}(\mathbf{K}^{x}_{s})^{-1}$$

Transformation Between Reference Frames

$$
{}^{x}\mathbf{K}^{y} = \begin{bmatrix} \cos(\theta_y - \theta_x) & -\sin(\theta_y - \theta_x) & 0 \\ \sin(\theta_y - \theta_x) & \cos(\theta_y - \theta_x) & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
\left({}^{x}\mathbf{K}^{y}\right)^{-1} = \left({}^{x}\mathbf{K}^{y}\right)^{T}
$$



- For example:

$$
\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_q^s \\ i_d^s \end{bmatrix}
$$

Space Vectors

- **Three-phase voltages**

$$v_{An}(t) + v_{Bn}(t) + v_{Cn}(t) = 0$$

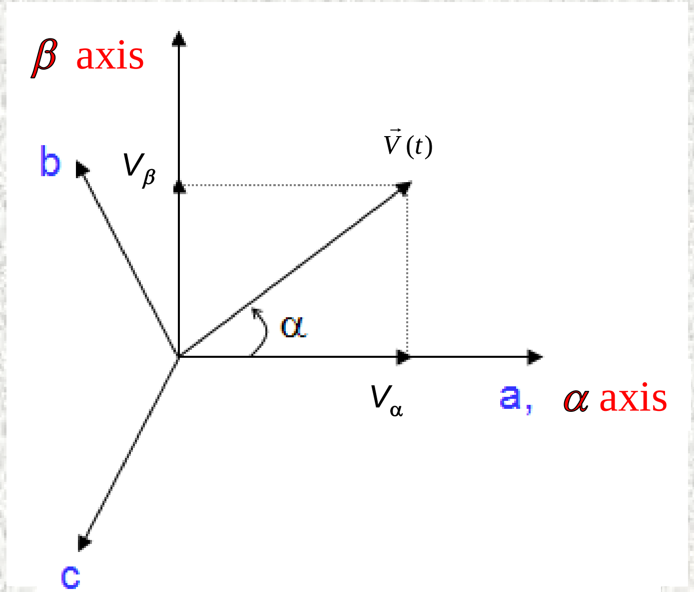
- **Two-phase voltages**

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos 0 & \cos\frac{2\pi}{3} & \cos\frac{4\pi}{3} \\ \sin 0 & \sin\frac{2\pi}{3} & \sin\frac{4\pi}{3} \end{bmatrix}\begin{bmatrix} v_{An}(t) \\ v_{Bn}(t) \\ v_{Cn}(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_{An}(t) \\ v_{Bn}(t) \\ v_{Cn}(t) \end{bmatrix}$$

- **Space vector representation**

$$\vec{V}(t) = v_\alpha(t) + j\, v_\beta(t)$$

$$\vec{V}(t) = \frac{2}{3}\left[v_{An}(t)e^{j0} + v_{Bn}(t)e^{j2\pi/3} + v_{Cn}(t)e^{j4\pi/3}\right]$$



where

$$\left|\vec{V}\right| = \sqrt{V_\alpha^{\,2} + V_\beta^{\,2}}$$

$$\alpha = \tan^{-1}\left(\frac{V_\beta}{V_\alpha}\right) = \omega_s t = 2\pi f_s t$$

(where, f_s = fundamental frequency)